MATH 147 QUIZ 1 SOLUTIONS

1. For the function $f(x,y)$, the point $(a,b) \in \mathbb{R}^2$, and the real number $L \in \mathbb{R}$, give the epsilon-delta definition for $\lim_{(x,y)\to(a,b)} f(x,y) = L$. (2 points)

   We say that $\lim_{(x,y)\to(a,b)} f(x,y) = L$ if for every $\epsilon > 0$, there exists a $\delta > 0$ such that $\|(x,y)-(a,b)\| < \delta$ implies that $|f(x,y) - L| < \epsilon$.

2. For $\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^6+2y^2}$, show that the limit along any line through the origin exists and equals zero, but if we take the limit along the curve $y = x^3$, the limit is not zero. What conclusion can you draw from this? (4 points)

   First we take the limit along any line through the origin. Let $y = ax$. Then, we have the limit

   $$\lim_{x\to 0} \frac{ax^4}{x^6 + 2a^2x^2} = \lim_{x\to 0} \frac{ax^2}{x^4 + 2a^2} = 0.$$

   On the other hand, if we approach along the curve $y = x^3$, we get

   $$\lim_{x\to 0} \frac{x^3x^3}{x^6 + 2(x^3)^2} = \lim_{x\to 0} \frac{x^6}{3x^6} = \lim_{x\to 0} 1/3 = 1/3 \neq 0.$$

   As the limits as we approach along different curves are not the same, we conclude that the limit itself must not exist.

3. Use polar coordinates to evaluate the limit $\lim_{(x,y)\to(0,0)} \frac{x^3+x^5}{x^2+y^2}$. (4 points)

   We make the standard polar substitutions $x = r\cos(\theta), y = r\sin(\theta)$, resulting in $r^2 = x^2 + y^2$. After this substitution, we have

   $$\lim_{r\to 0, \theta\in\mathbb{R}} \frac{r^3\cos^3(\theta) + r^5\cos^5(\theta)}{r^2} = \lim_{r\to 0, \theta\in\mathbb{R}} r(\cos^3(\theta) + r^3\cos^5(\theta)) = 0$$

   due to the squeeze theorem.